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HF CHANNEL DATA ERROR STATISTICS DESCRIPTION (I)

JUNE 1966

K. Brayer
O. Cardinale

Prepared for

DIRECTORATE OF AEROSPACE INSTRUMENTATION

ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts



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Project 705B
Prepared by
THE MITRE CORPORATION
Bedford, Massachusetts
Contract AF19(628)-5165

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FOREWORD

This report was prepared by the Range Communications Planning and Technology Subdepartment of The MITRE Corporation, Bedford, Massachusetts, under Contract AF 19(628)-5165. The work was directed by the Range Instrument Division under the Directorate of Aerospace Instrumentation, Air Force Electronics Systems Division, Laurence G. Hanscom Field, Bedford, Massachusetts. Captain Joseph J. Centofanti served as the Air Force Project Monitor for this program, identifiable as ESD (ESRI) Task 5932.07, Range Digital Data Transmission Improvement.

The authors wish to acknowledge the National Range Division for making available the operational facilities from which the HF data was drawn. The programming necessary for processing of the data was done by Mrs. Charlotte J. Saler, MITRE's Information Processing Department.

REVIEW AND APPROVAL

Publication of this technical report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.



C. V. HERRIGAN
Acting Director
Aerospace Instrumentation

ABSTRACT

A statistical description is presented of errors observed on a typical HF data transmission channel. The description includes the probability distribution functions for consecutive errors and error free gaps and their associated cross density statistics.

TABLE OF CONTENTS

	<u>Page</u>
LIST OF ILLUSTRATIONS	vi
LIST OF TABLES	vii
SECTION I HF CHANNEL DATA ERROR STATISTICS DESCRIPTION	1
INTRODUCTION	1
SECTION II DESCRIPTION OF CHANNEL STATISTICS	3
ORIGIN OF THE DATA	3
DATA PROCESSING METHODS	4
SECTION III DESCRIPTION OF CONSECUTIVE ERROR PROBABILITY	
DENSITY DISTRIBUTION	6
DESCRIPTION	7
SECTION IV CONSECUTIVE ERROR MODEL	8
MATHEMATICAL MODEL	8
APPLICATION OF MODEL OF DATA	9
APPLICATION OF CONSECUTIVE ERRORS	9
FREQUENCY OF OCCURRENCE OF SEQUENCES	10
SECTION V DESCRIPTION OF GAP DISTRIBUTION	12
DEFINITION	12
DESCRIPTION	13
SECTION VI CONCLUSIONS	15
APPENDIX I MATHEMATICAL DERIVATION OF MODEL FOR	
CONSECUTIVE ERRORS	16

LIST OF ILLUSTRATIONS

<u>Figure No.</u>		<u>Page</u>
1	Density Function of Multiple Errors Rate 1200 b/s, Class A X 10^{-5}	19
2	Density Function of Multiple Errors Rate 1200 b/s, Class A X 10^{-4}	20
3	Density Function of Multiple Errors Rate 1200 b/s, Class A X 10^{-3}	21
4	Density Function of Multiple Errors Rate 1200 b/s, Class A X 10^{-2}	22
5	Density Function of Multiple Errors Rate 2400 b/s, Class A X 10^{-5}	23
6	Density Function of Multiple Errors Rate 2400 b/s, Class A X 10^{-4}	24
7	Density Function of Multiple Errors Rate 2400 b/s, Class A X 10^{-5}	25
8	Density Function of Multiple Errors Rate 2400 b/s, Class A X 10^{-2}	26
9	Curve Fit to Consecutive Errors	27
10	Cumulative Distribution Function of Error Free Intervals, Rate 1200 b/s, Class A X 10^{-5}	28
11	Cumulative Distribution Function of Error Free Intervals, Rate 1200 b/s, Class A X 10^{-4}	29
12	Cumulative Distribution Function of Error Free Intervals, Rate 1200 b/s, Class A X 10^{-3}	30
13	Cumulative Distribution Function of Error Free Intervals, Rate 1200 b/s, Class A X 10^{-2}	31
14	Cumulative Distribution Function of Error Free Intervals, Rate 2400 b/s, Class A X 10^{-5}	32

LIST OF ILLUSTRATIONS (Concluded)

<u>Figure No.</u>		<u>Page</u>
15	Cumulative Distribution Function of Error Free Intervals, Rate 2400 b/s, Class A X 10^{-4}	33
16	Cumulative Distribution Function of Error Free Intervals, Rate 2400 b/s, Class A X 10^{-3}	34
17	Cumulative Distribution Function of Error Free Intervals, Rate 2400 b/s, Class A X 10^{-2}	35

LIST OF TABLES

<u>Table No.</u>		<u>Page</u>
I	Consecutive Error Statistics, Class = 10^{-5} , Rate 1200 b/s , 2400 b/s	36
II	Consecutive Error Statistics, Class = 10^{-4} , Rate 1200 b/s, 2400 b/s	37
III	Consecutive Error Statistics, Class = 10^{-3} Rate 1200 b/s, 2400 b/s	38
IV	Consecutive Error Statistics, Class = 10^{-2} Rate 2400 b/s, 1200 b/s	39
V	Error Free Interval Statistics, Class 10^{-5} Rate 1200 b/s	40
VI	Error Free Interval Statistics, Class 10^{-4} Rate 1200 b/s	42
VII	Error Free Interval Statistics, Class 10^{-3} Rate 1200 b/s	44
VIII	Error Free Interval Statistics, Class 10^{-2} Rate 1200 b/s	46

LIST OF TABLES (Concluded)

<u>Table No.</u>		<u>Page</u>
IX	Error Free Interval Statistics, Class 10^{-5} Rate 2400 b/s	48
X	Error Free Interval Statistics, Class 10^{-4} Rate 2400 b/s	50
XI	Error Free Interval Statistics, Class 10^{-3} Rate 2400 b/s	52
XII	Error Free Interval Statistics, Class 10^{-2} Rate 2400 b/s	54

SECTION I

HF CHANNEL DATA ERROR STATISTICS DESCRIPTION (I)

INTRODUCTION

The purpose of this report is to present the results of one phase of the Range Digital Data Transmission Improvement program being conducted at MITRE. The scope of this report is limited to the presentation of selected statistics describing the error occurrences on a typical long range HF data transmission link.

Part of the Range Digital Data Transmission Improvement program is concerned with the reduction of errors in long range data transmission links that are now in operation, using HF radio channels as part of the transmission link. One technique which appears to show considerable promise in achieving significant error reduction in data transmission, is the forward error detection and correction technique. However, in order to intelligently determine the performance requirements for equipment using this technique, a detailed knowledge of some of the statistical characteristics of the errors introduced in the data transmission channel by the HF medium is required.

The statistics presented here are designed to provide meaningful information that is useful in the design of block type error correction equipment implementation for improvement of channel error rate. These descriptive statistics are based on observation and measurement of channel errors recorded during the 1965 Antigua-Ascension HF data link tests under conditions that are typical for that channel.

A detailed description of the test implementation is given in MTR-2, dated October 2, 1965, entitled 'Test Plan-1965 Antigua-Ascension High Frequency Tests, by Brayer, Greim and Nelson. The channel implementation consisted of a transequatorial path of approximately 3,200 miles one way or a total of 6,400 miles for the full loop test path. The RF facilities were provided by the Air Force Eastern Test Range. The digital modems used for this test were leased from Collins Radio Company. The tests were organized and performed by MITRE Corporation.

A total of 151 runs of data were used to generate the descriptive channel error statistics presented herein. This includes 121 runs of data taken at 1200 bits per second and 30 runs of data taken at 2400 bits per second. Each run of data represents a continuous test transmission period with an average length of ten minutes.

This report presents one aspect of the results of the processing that was performed on the recorded data, namely. the consecutive error probability distributions and associated cross-density statistics, and the error-free gap probability distributions and associated cross-density statistics.

SECTION II

DESCRIPTION OF CHANNEL STATISTICS

ORIGIN OF THE DATA

The data which represents the effects of the channel on data transmission quality was derived from an HF link between Antigua and Ascension Island. This test data was gathered over a period of time which represents the effects of diurnal variations on the channel as well as short term effects such as fading. The data used herein was gathered over a six week period of time.

The error data was obtained in the following manner. A test message was transmitted over the transequatorial path and relayed back without regeneration to the originating station (Antigua Island). A comparison was made of the transmitted and received test messages to determine the errors that were introduced by the channel. The comparison was performed using modulo two addition of the received test message and the transmitted test message which was suitably delayed to account for the total transmission path delay. The modulo two adder output provided a 'one' for each bit received in error and a 'zero' for each bit received correctly. The output of the modulo two adder was recorded on magnetic tape in real time. The relevant channel conditions such as noise burst and interference signals were also monitored and recorded. There are no channel errors that were recorded due to any channel outages caused by equipment failures.

The modem used for these tests was the Collins Radio Company model TE-216 modem. This unit is designed for digital data transmission over HF and wire line facilities. The modem is designed to operate at data rates up to 3,600 bits per second. The modulation used is four level phase modulation of each of 16 frequency multiplexed audio tones for 2400 bits per second data rate. Only eight tones are used for 1200 bits per second data rate. The phase of each tone is changed at a rate of 75 times per second. The detection and encoding technique used in the modem permits a bit rate of 150 bits per second per tone. This is accomplished using differential phase shift keying and detection. Differential coherent phase shift detection is used to decode the phase modulated signal for each incoming tone. A detailed description of the modem theory of operation is given in the Collins Radio Company Instruction Manual on the TE-216 modem.

The recorded data was processed at The MITRE Corporation Computing Facilities and the data was segregated into the following two classes, namely, consecutive errors and gaps. The consecutive error interval is always immediately preceded and followed by a correct bit. The error free gap is always immediately preceded and followed by a bit in error. There are no errors permitted within the gap region, and no correct bits in the consecutive error interval.

DATA PROCESSING METHODS

The magnetic tape recording of errors in the test transmissions were made in the field and then transported to The MITRE Corporation Bedford Facilities. Here the data was processed through magnetic tape conversion equipment which provided IBM compatible tapes of the recorded errors. In this manner the data was placed in a format which permitted efficient application of high speed data processing techniques.

The raw data from these runs was processed at The MITRE Computation Facility in the following manner. The data runs were first sorted into separate decades of error rate from 10^{-2} to 10^{-5} , then each decade was separated into the two data rates (1200 bits per second and 2400 bits per second). The data was separated into non-overlapping decades to reduce the total computer time required, while obtaining meaningful results. Instead of 302 individual distribution functions (two for each run of data) the data has been reduced to a total of 16 distribution functions. The distribution functions calculated are as follows:

- a) Mean cumulative distributions function of gaps,
- b) Mean probability density functions of consecutive errors.

In order to account for the contributions of individual data runs in each decade, the following functions were also tabulated for each gap size and consecutive error interval in each decade.

- a) Means of each gap size and consecutive error interval,
- b) Medians of each gap size and consecutive error interval
- c) Standard deviation of each gap size and consecutive error interval
- d) Range (extreme values) of each gap size and consecutive error interval.

The calculations were performed, in the case of the mean (for each class of transmission rate and decade of error rate) by taking all points from the probability density functions corresponding to a given abscissa and averaging these points together to find the sample mean.

For example, a sample mean for a selected value of consecutive errors is calculated as follows. A class of data is selected, for example 1200 bits per second data within the 10^{-5} decade of error rate. The normalized frequency of occurrence of 'n' consecutive errors in each run of the selected data class is calculated, and the corresponding mean value is tabulated for each value of 'n'. This was done for each value of consecutive errors as shown in figures 1 through 8. The other statistics were found using the same procedure was applied to each statistic. The obtained mean for each point was then plotted to generate the probability density and distribution functions. Since runs of different time length were averaged together each run was normalized on a minute basis. The procedure of generating the mean density functions (cumulative distribution functions) in this manner is permissible since the combination of 'n' density functions (each of which sums to unity) and their corresponding means is identical to the sum of the means of the sub-points which generate each distribution¹.

Since these different runs were drawn from the same class sample population the process of averaging the statistics for each gap of multiple error size and then summing, will generate a mean probability distribution function of the data. These curves and corresponding tabulations are presented in the following sections.

¹ The sum of the means is the mean of the sums.

SECTION III

DESCRIPTION OF CONSECUTIVE ERROR PROBABILITY DENSITY DISTRIBUTION

DEFINITION

The consecutive error interval is defined as a group of bits, all of which are in error. The interval is immediately preceded by a correct bit and terminated by a correct bit. The consecutive error intervals were recorded in unit steps up to and including six consecutive errors. In the data that was analyzed, it was found that the occurrence of data groups with more than six consecutive errors was a rare event compared to the occurrence of errors.

The consecutive error probability density function is defined as the probability of occurrence of N consecutive errors where N is any positive integer.

A typical probability density function of consecutive error occurrence is shown in Figure 6. The probability of error is presented in percentage form. The plotted data indicates that 94% of the errors observed were followed by a correct digit and 5% of the errors observed were followed by an incorrect digit then a correct digit. The following relationship holds true for each class of observed data:

$$\sum_{x=1}^{\infty} p \{x\} = 100\%$$

where 100% corresponds to a total probability of unit. If this probability density distribution corresponds to a class of data of average error rate 10^{-2} , then it can be determined directly from the corresponding probability density distribution that:

$$p \{\text{single error}\} = .94 \times 10^{-2}$$

$$p \{\text{double error}\} = .05 \times 10^{-2}$$

DESCRIPTION

The consecutive error probability density functions are shown in Figures 1 through 8¹ inclusive. A separate probability density curve has been plotted for each decade of average error rate. The curves are further differentiated into those decades that are associated with the 1200 bits per second data and the remaining data was 2400 bits per second. In reading these functions it should be remembered that these are discrete functions and only the points charted can be correctly read. Interpolation can be made for uncharted discrete points but noninteger values should not be read as they have no physical significance.

The plotted data shows that the probability of any value of N consecutive errors is practically independent of the average error rate for both the 1200 and 2400 bits per second data. The probability density functions presented herein have been truncated above six consecutive errors, since the occurrence of more than six consecutive errors in the observed data was a rare event.

In addition to the mean values of the distribution, the following statistical parameters of the data for each point are tabulated in Table I to IV; the median, standard deviation, and the range for each data point. These parameters are presented for the six consecutive error intervals as a separate tabulation.

¹

See Papoulis - Probability, Random Variables and Stochastic Processes, McGraw-Hill 1965 for description of data presentation format.

SECTION IV

CONSECUTIVE ERROR MODEL

A large number of graphs and charts of experimental data has been presented here. It is advantageous to have a mathematical model which can generate valid statistics which fit the experimental data in general. This model can then be used to obtain similar data from just a few measured parameters of the channel. The proposed model which is valid for occurrence of consecutive errors only is described below.

MATHEMATICAL MODEL

A mathematical model is adopted to obtain the occurrence of consecutive errors and additionally the probability of occurrence of sequences of consecutive errors or error-free digits.

The results are that

$$O_c \{e_c^n\} = \left[\prod_{i=1}^{n-1} \frac{a-i}{a+b-i} \right] \left[1 - \frac{a-(j)}{a+b-(j)} \right]; \quad 1 < n \leq a, \quad j = i_{\max} + 1$$

where O_c represents occurrence

e represents errors

c represents correct bits

a equals number of errors in data

n is the number of consecutive errors

$$\text{and } \Pr \{U = u = 2v\} = 2 \frac{\binom{a-1}{v-1} \binom{b-1}{v-1}}{\binom{a+b}{b}} \quad \text{for even values of } U$$

Where U represents a set of random variables denoting the occurrence of

u sequences of digits of the same type

u is a member of the set of random variables

and

$$\Pr \{U = v = 2v + 1\} = \frac{\binom{a-1}{v} \binom{b-1}{v-1} + \binom{a-1}{v-1} \binom{b-1}{v}}{\binom{a+b}{b}}$$

for odd values of u

APPLICATION OF MODEL OF DATA

Suppose we associate the type a objects with errors and the type b objects with correct bits in digital data transmission. A probability function can be used to get the probability function for consecutive errors, and the probability of frequency of occurrence of such sequences.

APPLICATION TO CONSECUTIVE ERRORS

We consider the following formulas for the occurrence of consecutive errors.

Type

$$e_{ec} \quad \frac{a}{a+b} \times \frac{a-1}{a+b-1} \times \left(1 - \frac{a-2}{a+b-2}\right)$$

$$e_{eec} \quad \frac{a}{a+b} \times \frac{a-1}{a+b-1} \times \frac{a-2}{a+b-2} \times \left(1 - \frac{a-3}{a+b-3}\right)$$

$$e_{ec}^n \quad \left[\prod_{i=j}^{n-1} \frac{a-i}{a+b-i} \right] \left[1 - \frac{a-j}{a+b-j} \right]; \quad 1 < n \leq a, \quad j = i_{\max} + 1$$

In Figure 9 an actual distribution of data is presented which had an error rate of 10^{-4} . The theoretical model distribution is also presented in this figure. It is evident from examination of the density functions presented that a very good fit is obtained.

FREQUENCY OF OCCURRENCE OF SEQUENCES

Given that the average error rate is X . There are a errors and b correct bits.

$$X = \frac{a}{a + b}$$

The probability of occurrence of sequences U less than or equal to u is given by the formulas previously defined. For example, if we have 24 bits and three possible errors, the probability of three consecutive errors is the probability of two sequences within the 24 bits plus the probability of three sequences where all the errors are in one internal sequence.

$$\Pr \{U = 2\} + \Pr \{U = 3, cc \dots ee \dots cc\} = 2 \frac{\binom{3-1}{1-1} \binom{21-1}{1-1}}{\binom{24}{3}} +$$

$$\frac{\binom{3-1}{1-1} \binom{21-1}{1-1}}{\binom{24}{3}}$$

$$= 2 \frac{\binom{2}{0} \binom{20}{0} + \binom{2}{1} \binom{20}{0}}{\binom{24}{3}}$$

$$= 2 \frac{\frac{2!}{0! 2!} \times \frac{20!}{0! 20!}}{\frac{24!}{3! 21!}} + \frac{\frac{2!}{1! 1!} \times \frac{20!}{0! 20!}}{\frac{24!}{3! 21!}}$$

$$= \frac{1}{\frac{24!}{3! 22!}} (2 + 2) = \frac{4 \times 3! \times 21!}{24!}$$

$$= \frac{4 \times 3 \times 2}{24 \times 23 \times 22} = \frac{2}{1012} = \frac{1}{506} \approx 2 \times 10^{-3}$$

As a comparison the data was subdivided into 24 bit blocks and processed to determine the relative occurrence of triple errors. The result was that on a relative frequency basis, the occurrence was 3.7×10^{-3} for the total probability of occurrence of [eee cc] and [ccc ... ceeec ... c].

SECTION V

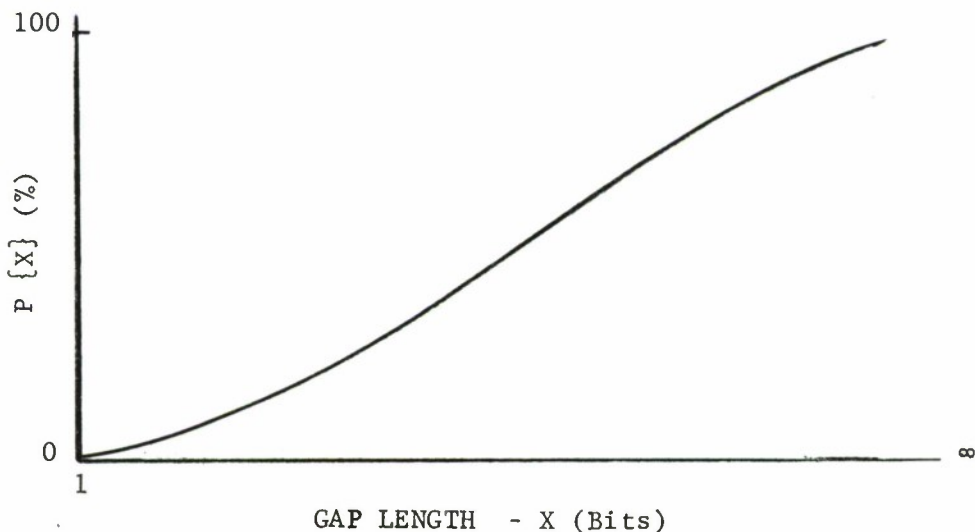
DESCRIPTION OF GAP DISTRIBUTION

DEFINITION

The gap is defined as an interval of consecutive correct bits which are immediately preceded and terminated by a bit in error. The gaps are recorded in unit steps up to 32 bits and thereafter in powers of two, i.e., 64, 128, 256 and etc. up to a maximum length of 8,192 bits. The gaps of length greater than 32 and less than 64 for example, are lumped under the single classification of 64 bits.

The gap distribution function is defined for each value of gap length X , as the probability of the occurrence of a gap of length X bits or less. The maximum gap length, or course, would be the total number of bits in the run. This would be approximately 720,000 bits for a 1200 bit per second run and 1,440,000 bits for a 2400 bits per second run.

A typical cumulative distribution function for error-free intervals is presented below.



Error-free intervals can range from the gap of length zero to an infinitely long interval. As a notational convenience (since a semi-log scale is used) the non-error-free case is taken as size one, and each gap size beyond this is also increased by one.

From this cumulative curve, the following values are read:

$$P \{1\} = p \{ee\}$$

$$P \{2\} = p \{ee\} + p \{eoe\}$$

$$P \{3\} = p \{2\} + p \{eooe\}$$

etc.

In this way, it is possible to obtain the probability of the occurrence of situations of the form $eo^n e$. The curve is truncated at 10,000 for which more than 90% of the information is included.

It is also important to have the statistics from which the points on the mean distribution were obtained. These values have thus been obtained as they were in the case of consecutive error functions.

As in the case of consecutive error distributions, the distribution functions for error-free intervals are presented along with tabulations cross-distribution statistics for each class of data. It should also be remembered that these are also discrete probability functions.

DESCRIPTION

The gap distribution functions are shown in Figures 10 through 17 included herein. A separate distribution curve has been plotted for each decade of average error rate. For example, the curve in Figure 10 represents all the data that was derived from the 1200 bits per second runs that have average error rates of $A \times 10^{-5}$ where $1 < A \leq 10$ which is called the 10^{-5} decade. The gap distribution functions are shown for the same decades, for the 2400 bits per second data. The abscissa shown for each gap size is the actual gap plus one. For example, the abscissa value of 2 corresponds to an actual gap of size 1. This procedure was used for convenience to permit the use of semilog type plots. When the gaps exceed length 32, the values of gaps were incremented in powers of 2 for convenience in presenting the data. Thus, the gaps of size 64 actually includes all the gaps from 33 to 64 inclusive.

In addition to the mean values of the distributions, the following statistical parameters of the data for each point are tabulated in Tables V to XII. These parameters are the median, standard deviation, and the range for each data point.

When the cumulative gap distributions are examined, it will be noted that the distribution curve is progressively lower for short gap lengths as the error rate is decreased. This is to be expected, since the lower the error rate, the longer error free gaps should occur more frequently and the shorter gap lengths less frequently.

The gap distribution functions for each decade except 10^{-5} show that approximately 90% or more of all gaps are less than 8,192 bits long. For the case of the 10^{-5} decade, the error rate is so low that a considerable number of gaps are greater than 8,192 bits long (approximately 45%).

SECTION VI

CONCLUSIONS

The frequency of consecutive errors observed in the recorded test data indicates that approximately 95% of the errors that occurred were single errors, that is, a bit in error immediately preceded and followed by correct bits. Thus, treatment of this data from the point of view of consecutive error bursts is of little value.

The mathematical model developed for the observed consecutive error distribution indicates that these errors are fairly well behaved functions and can be closely approximated by a model which assumes independence in consecutive errors.

Analysis of the cumulative gap distribution curves derived for the observed data shows that the frequency of occurrence of large gaps consistently increases with improvement in the average error rate. These curves also show a consistent discontinuity or sudden rise at lengths 16 and 32. This behavior is consistent with the observed effects of interference on a tone of the modem output.


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APPENDIX I

MATHEMATICAL DERIVATION OF MODEL FOR CONSECUTIVE ERRORS

Suppose we have "a" elements of type A and "b" of type B, $a + b = N$ and these N elements are chosen randomly one at a time and no replacement is permitted. The number of sequences of like elements S_a, S_b have a distribution. This is the distribution that shall be derived. As a starting point, it is necessary to consider two problems in combinatorial analysis. The first is to find the number of ways in which r identical objects are placed in n number slots. This will be considered in two ways; one with the requirement that an unlimited number (S_r) of objects can be placed in any slot and the other that no slot be empty. Consider a row of n slots with r objects to be placed in them. The n slots can be shown between $n + 1$ bars, e.g.

| A A | | A | | |

represents 5 slots with 2, 0, 1, 0 and 0 objects A in the slots. Of the $n + 1$ bars two must be of the ends and $n-1$ can be anywhere. The number of ways of filling n slots is the number of arrangements of $n-1$ objects of one type and r of the other type and is thus

$$\frac{[(n - 1) + r]!}{(n - 1)! r!}$$

If now the number of objects is greater than the number of slots ($r > n$) no slot will be permitted to be empty. Thus in our example there cannot be two adjacent bars as this would be an empty slot. Of the $r-1$ spaces between the r objects we can choose $n-1$ as bar locations. This choice can be made in

$$\binom{r-1}{n-1} = \frac{(r-1)!}{(n-1)! (r-n)!}$$

ways. This is the number of ways in which r objects can be placed in n slots ($r \geq n$) with no empty slots.

It is now possible to find the probability of U sequences, each having like elements i.e. $\Pr \{U = u\}$ for "a" objects of one kind and "b" objects of the other kind. The total number of arrangement is

$$\binom{a+b}{a} \triangleq \binom{a+b}{b}$$

If all arrangements are equally probable

$$\Pr \{U = u\} = \frac{\text{arrangements given } u \text{ sequences}}{\text{all arrangements}}$$

Consider the case for $u = \text{even}$. Let $u = 2v$. From the previous theory we can fill the v slots which are associated with the "a" objects in

$$\binom{a-1}{v-1} \text{ ways}$$

Likewise the second v which are associated with the "b" objects are fitted in

$$\binom{b-1}{v-1} \text{ ways}$$

Now for u even if there is a sequence $S_a S_b S_a S_b$ it is equivalent to $S_b S_a S_b S_a$ in terms of numbers of sequences. Thus there are

$$\binom{a-1}{v-1} \binom{b-1}{v-1} \text{ arrangements}$$

$$\Pr \{U = u = 2v\} = \frac{\binom{a-1}{v-1} \binom{b-1}{v-1}}{\binom{a+b}{a}}$$

If u is odd. Let $u = 2v + 1$.

By similar reasoning

$$\Pr \{U = u = 2v + 1\} = \frac{\binom{a-1}{v} \binom{b-1}{v-1} + \binom{a-1}{v-1} \binom{b-1}{v}}{\binom{a+b}{a}}$$

Where sequences $S_a S_b S_a$ are drawn as

$$\binom{a-1}{v-1} \binom{b-1}{v}$$

and sequences S_b S_a S_b are drawn as

$$\binom{a-1}{v} \binom{b-1}{v-1}$$

Thus a probabilistic model is developed for the number of sequences of two kinds of elements which occurs assuming an independent, random distribution.

1B17,900

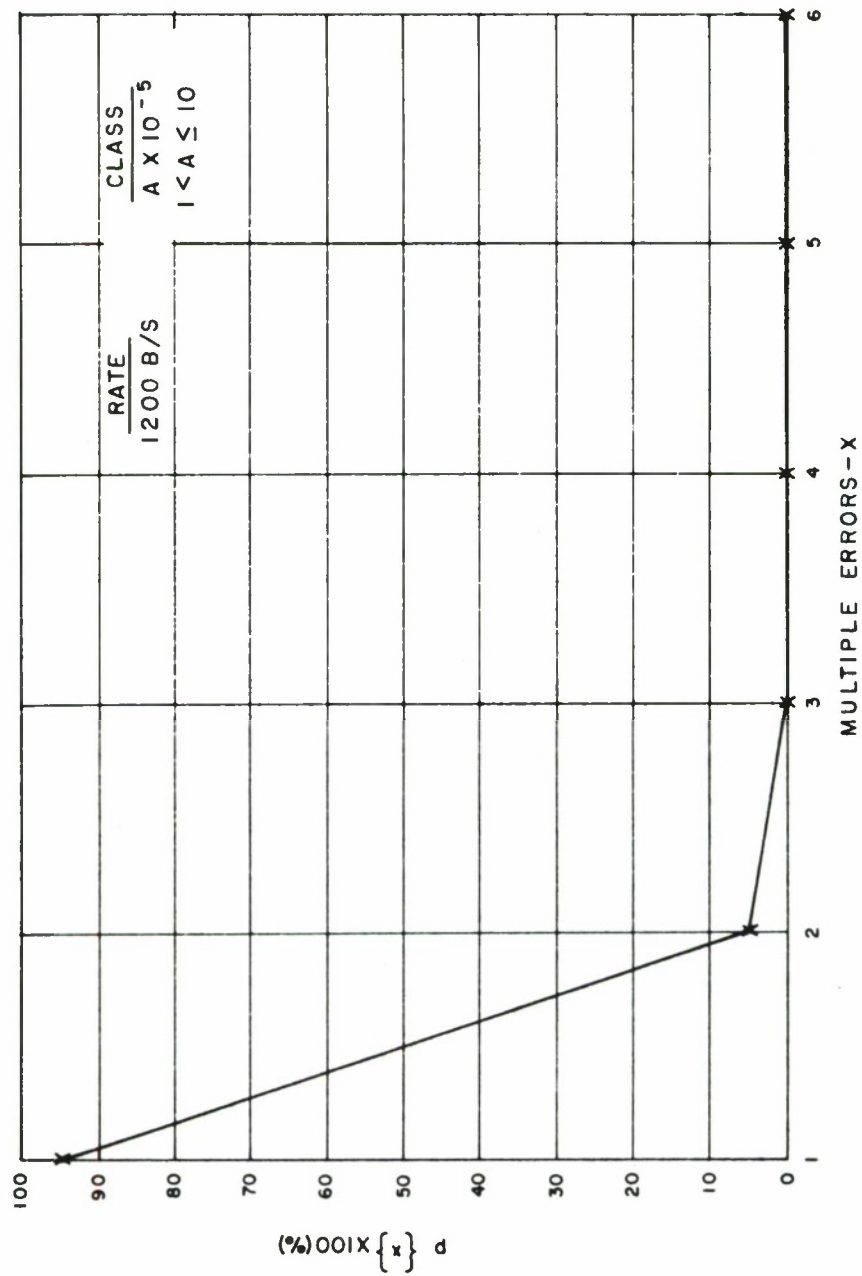


FIG 1 DENSITY FUNCTION OF MULTIPLE ERRORS

IB17,902

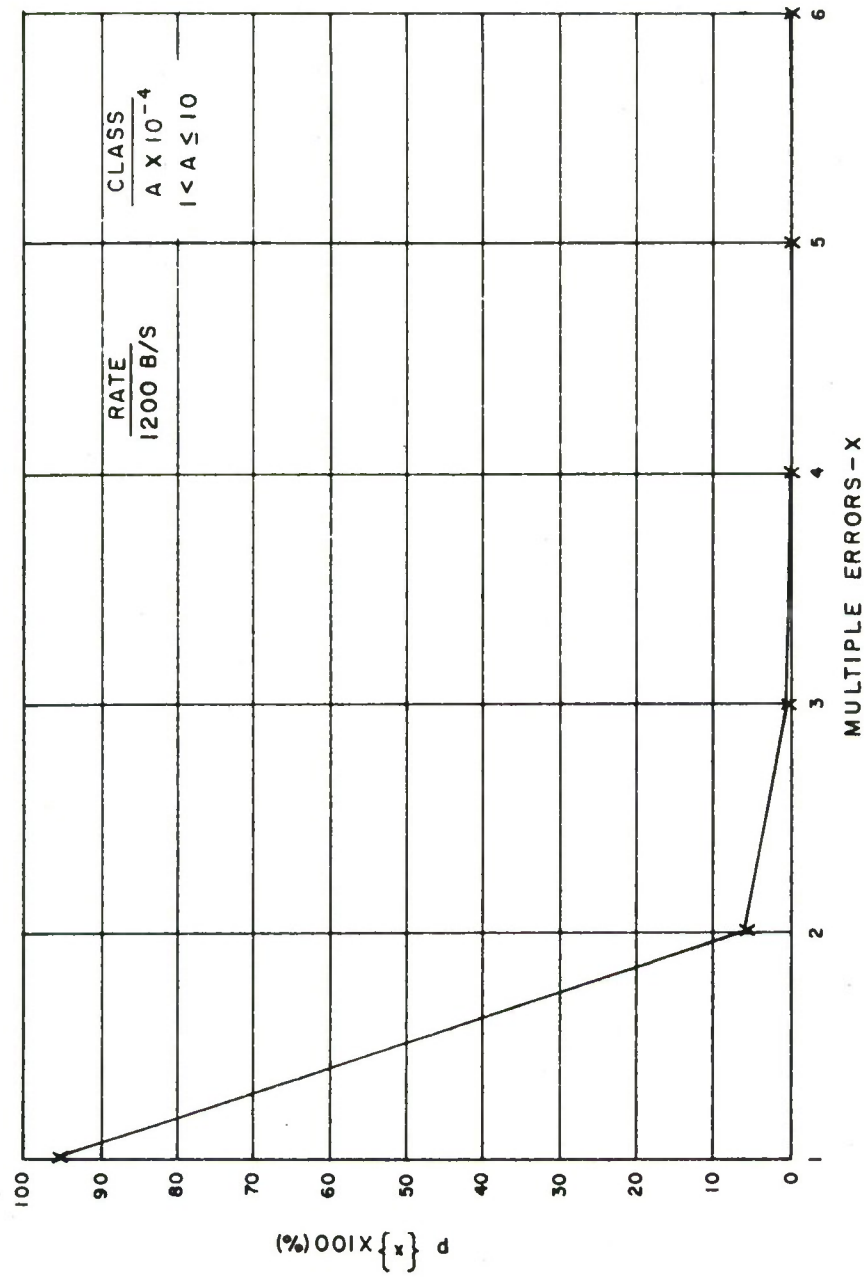


FIG. 2 DENSITY FUNCTION OF MULTIPLE ERRORS

IB17,901

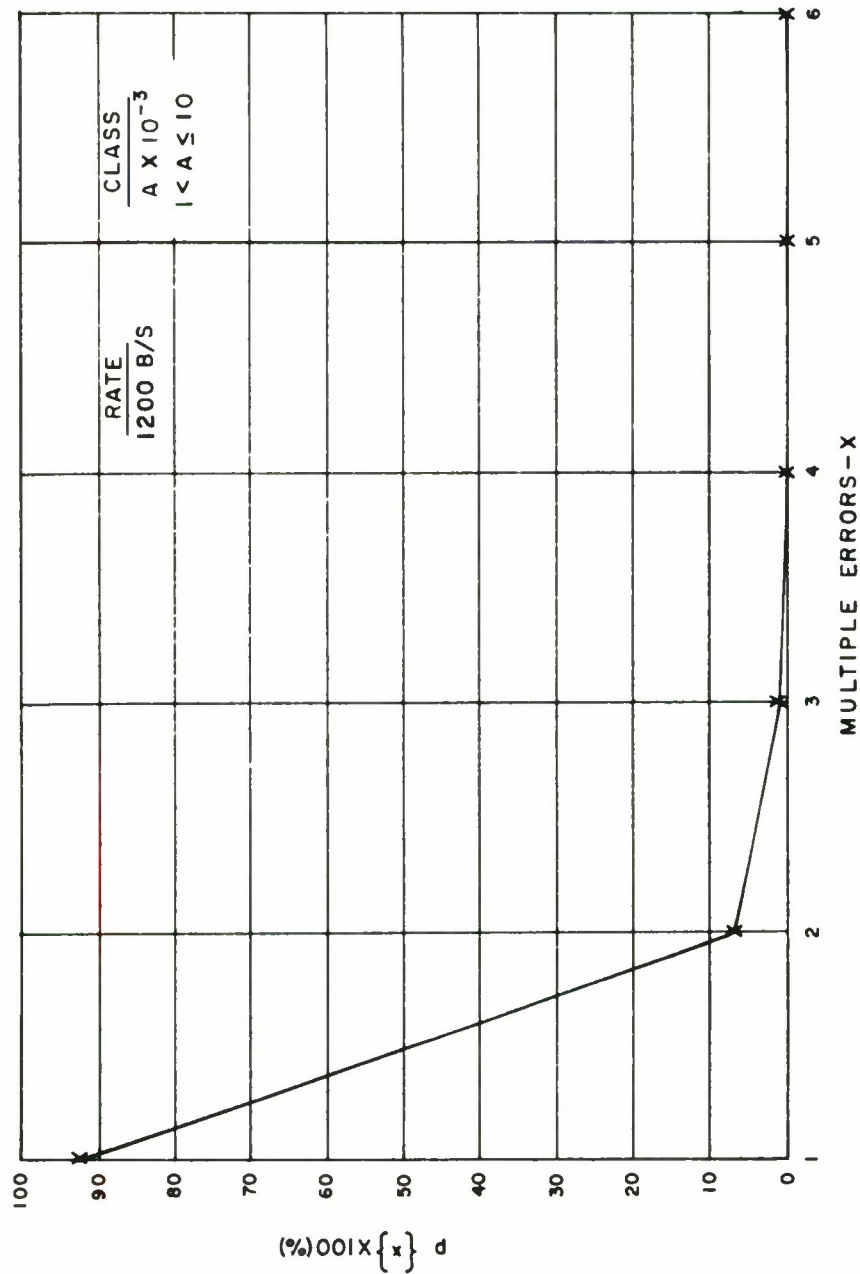


FIG. 3 DENSITY FUNCTION OF MULTIPLE ERRORS

1B17,903

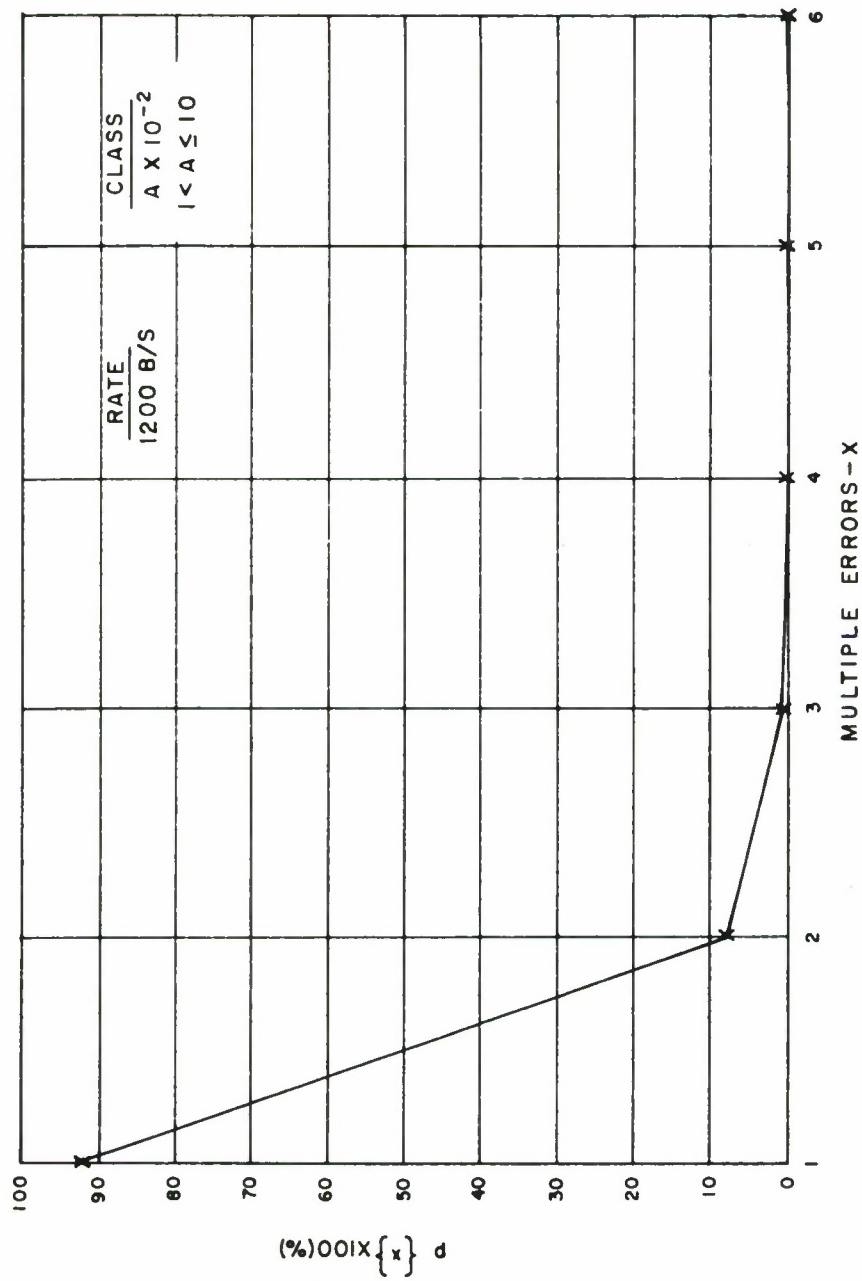


FIG. 4 DENSITY FUNCTION OF MULTIPLE ERRORS

FIG. 5 DENSITY FUNCTION OF MULTIPLE ERRORS

1817,898

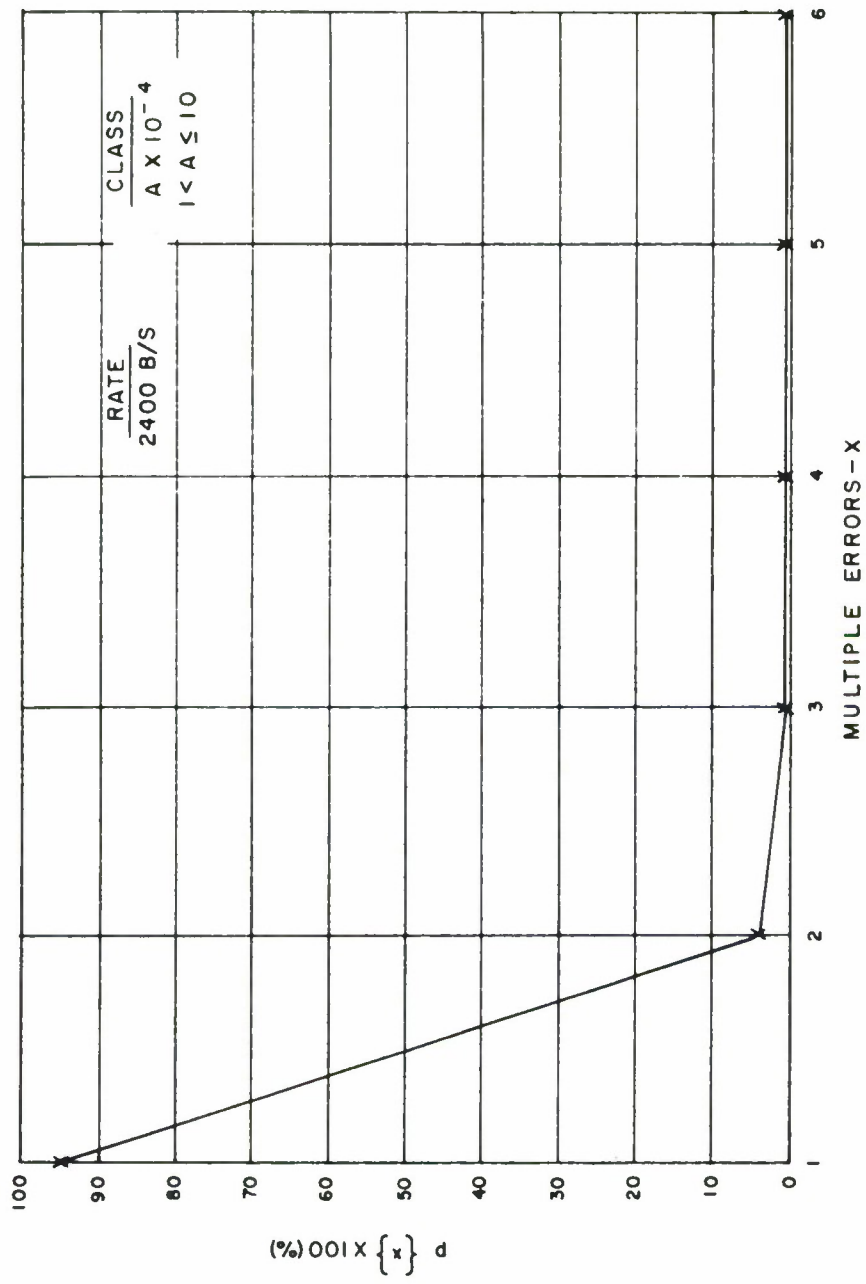


FIG. 6 DENSITY FUNCTION OF MULTIPLE ERRORS

18 17,898

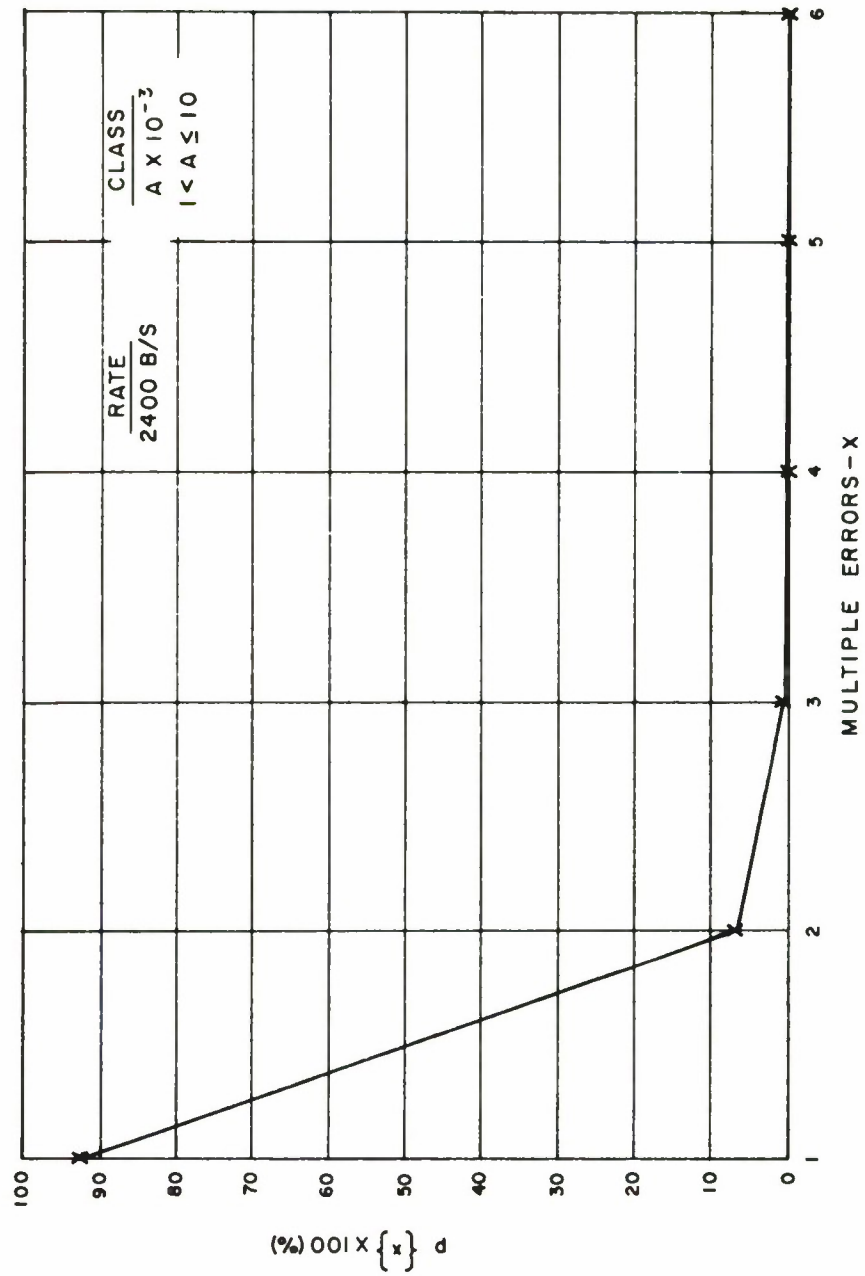


FIG. 7 DENSITY FUNCTION OF MULTIPLE ERRORS

FIG. 8 DENSITY FUNCTION OF MULTIPLE ERRORS

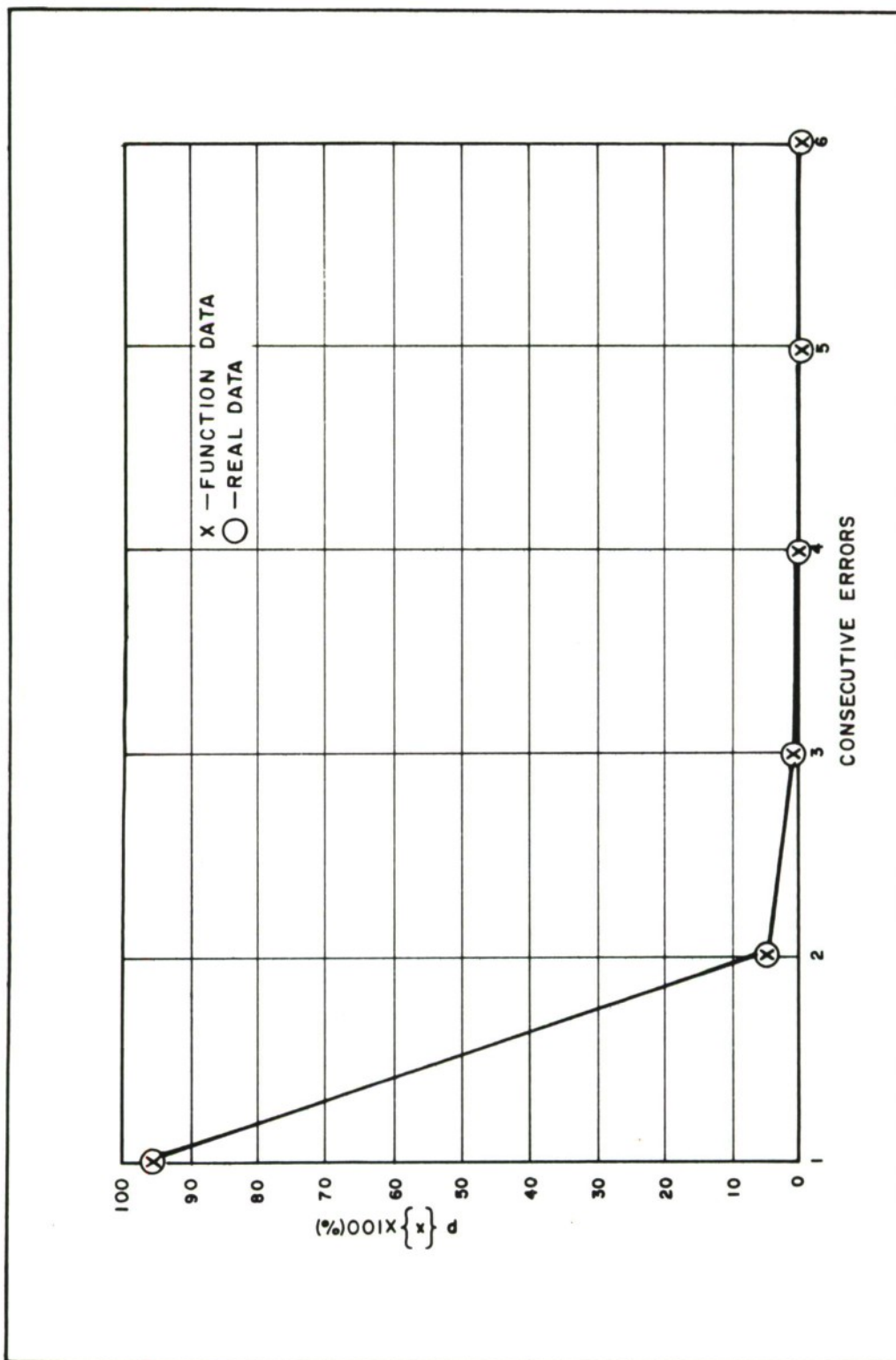


FIG. 9 - CURVE FIT TO CONSECUTIVE ERRORS

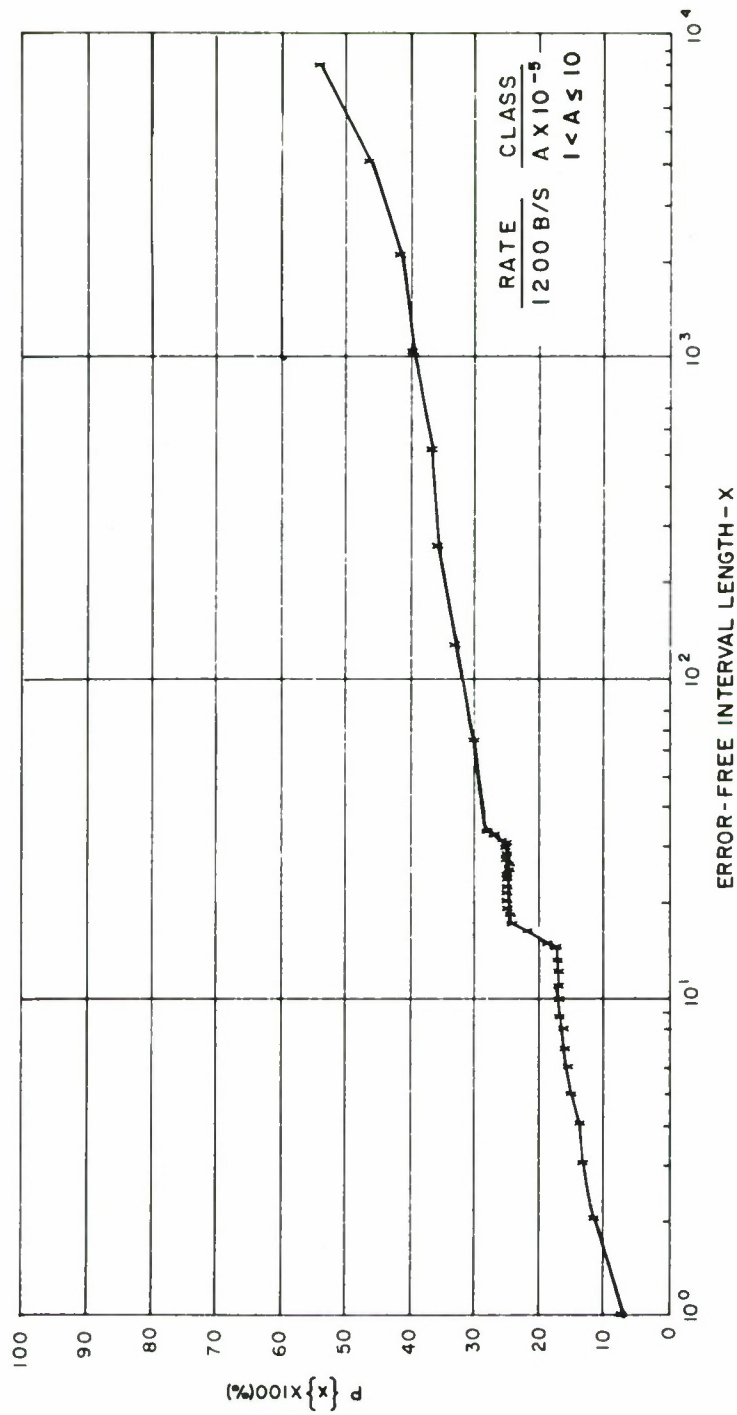


FIG.10 CUMULATIVE DISTRIBUTION FUNCTION OF ERROR FREE INTERVALS

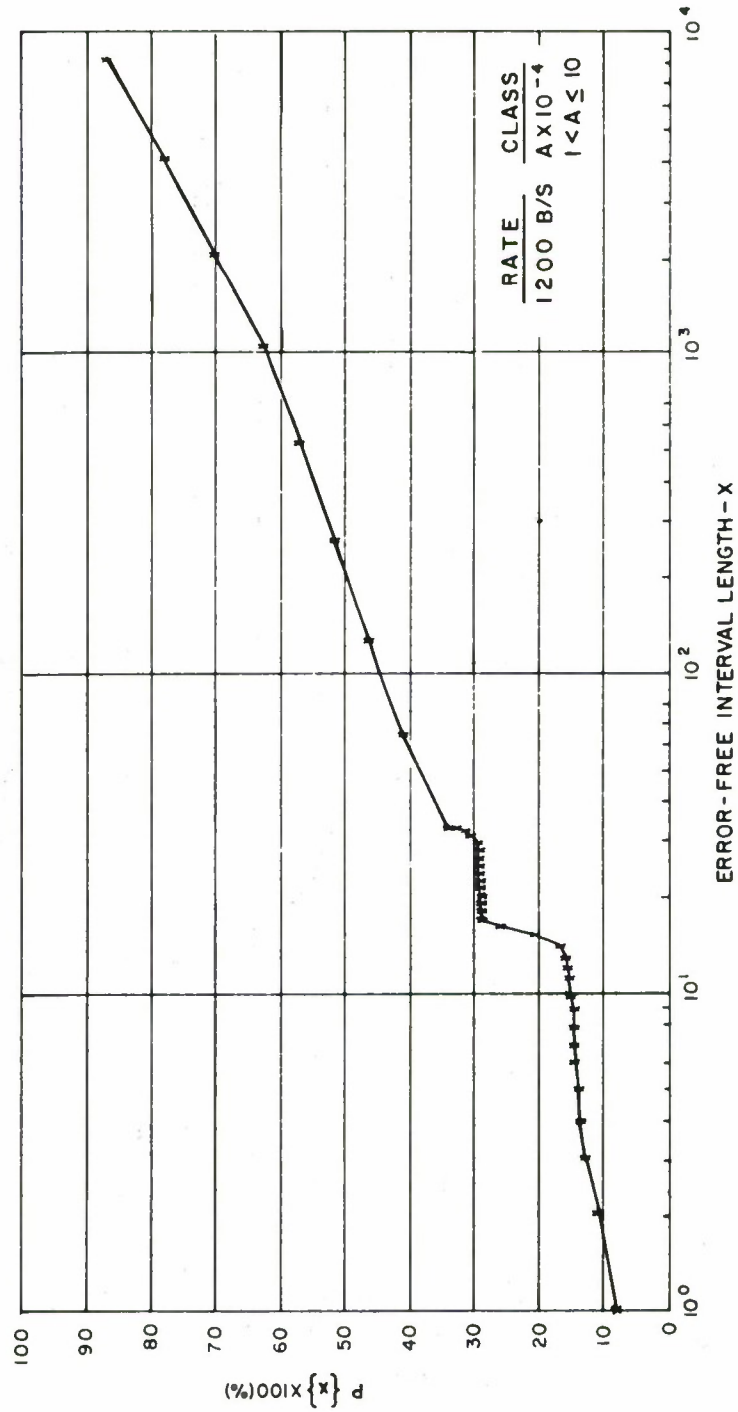


FIG. II CUMULATIVE DISTRIBUTION FUNCTION OF ERROR FREE INTERVALS

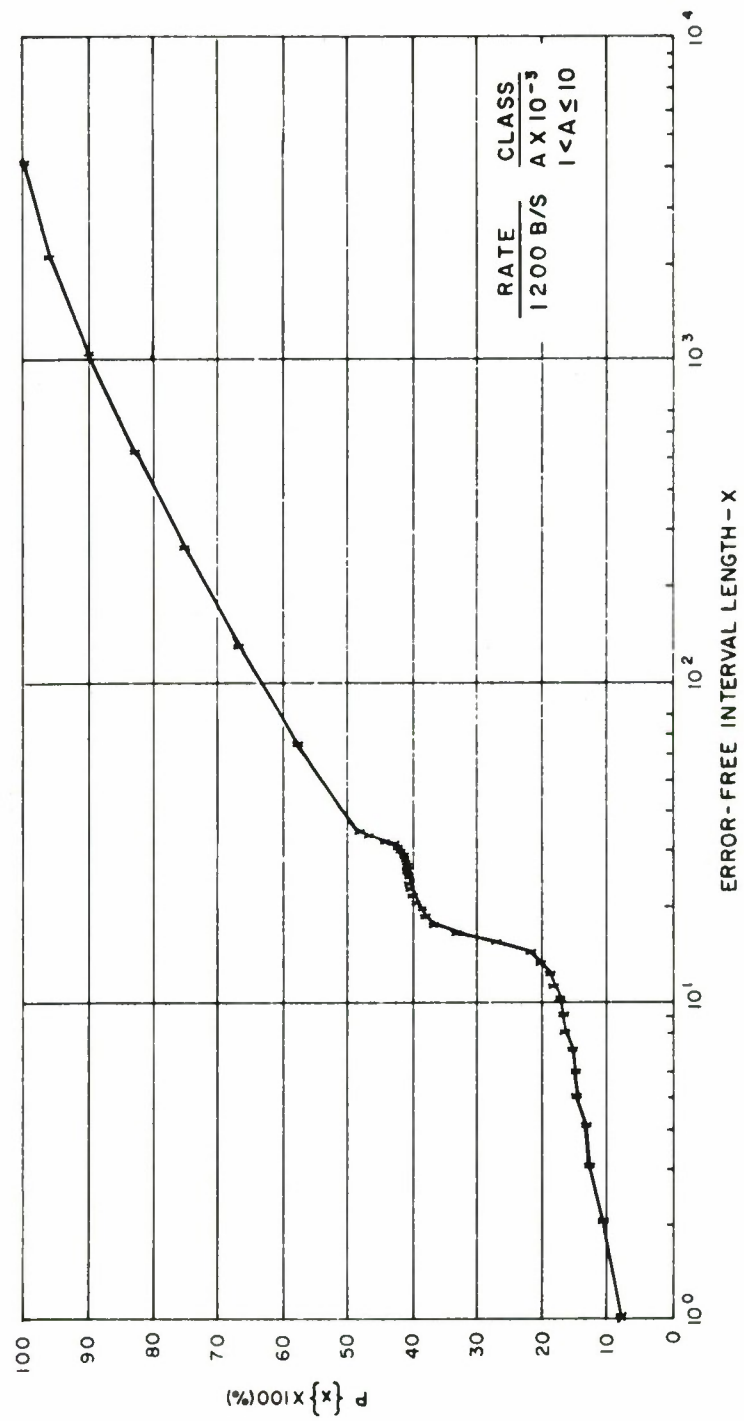


FIG.12 CUMULATIVE DISTRIBUTION FUNCTION OF ERROR FREE INTERVALS

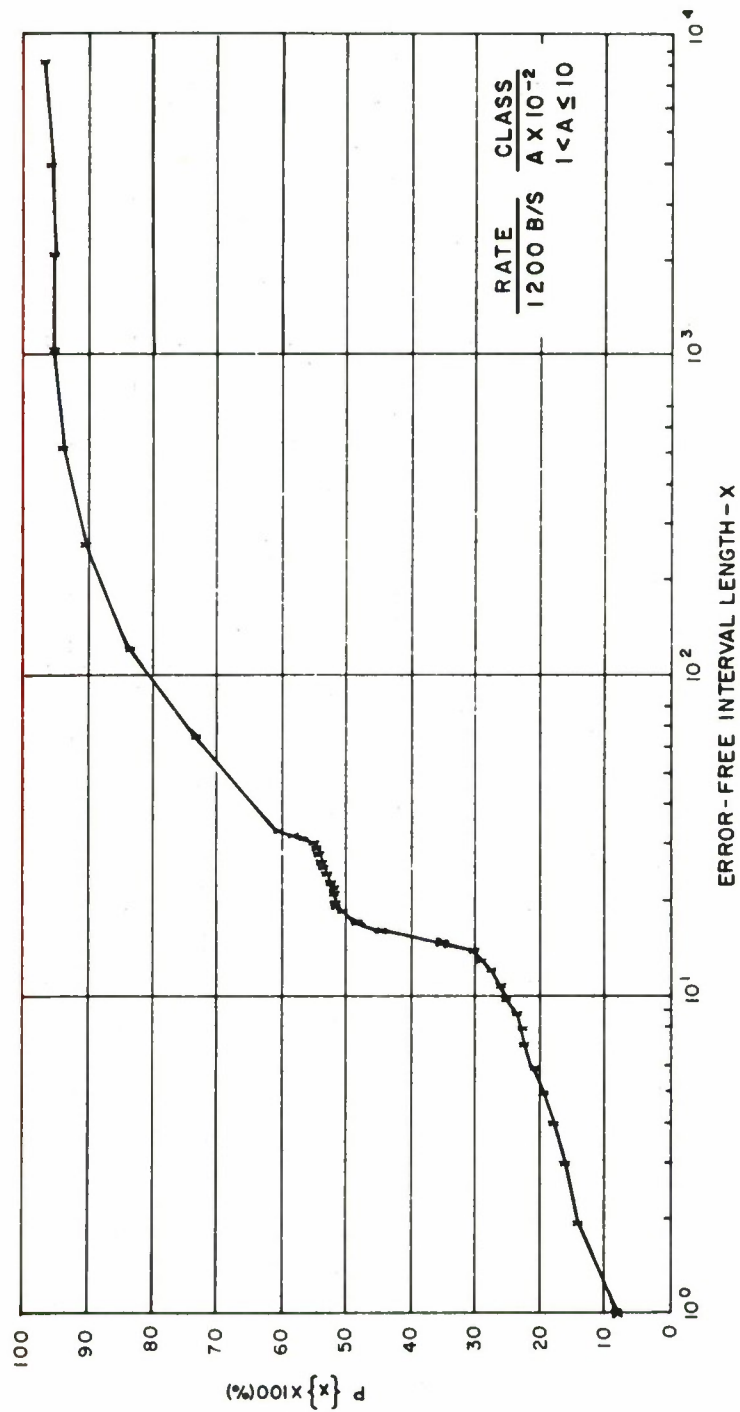


FIG.13 CUMULATIVE DISTRIBUTION FUNCTION OF ERROR FREE INTERVALS

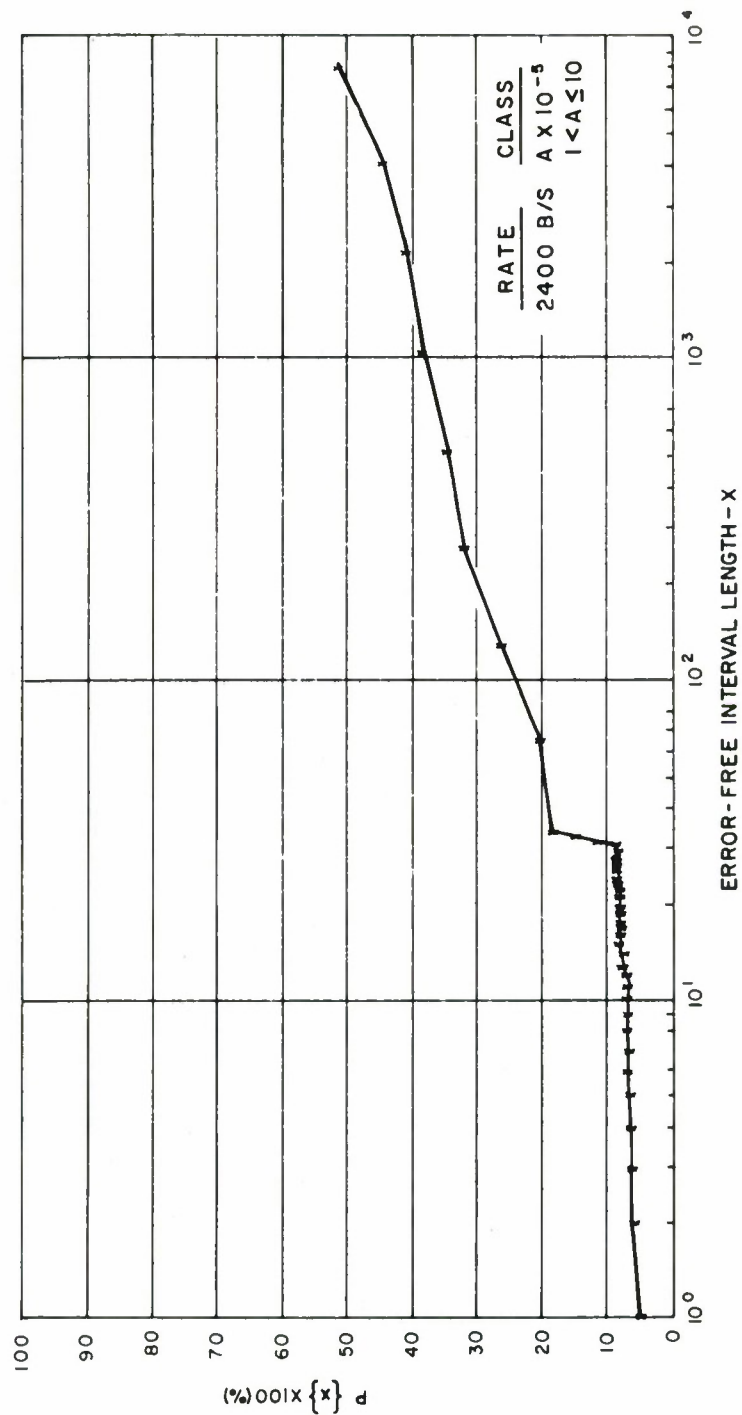


FIG. 14 CUMULATIVE DISTRIBUTION FUNCTION OF ERROR FREE INTERVALS

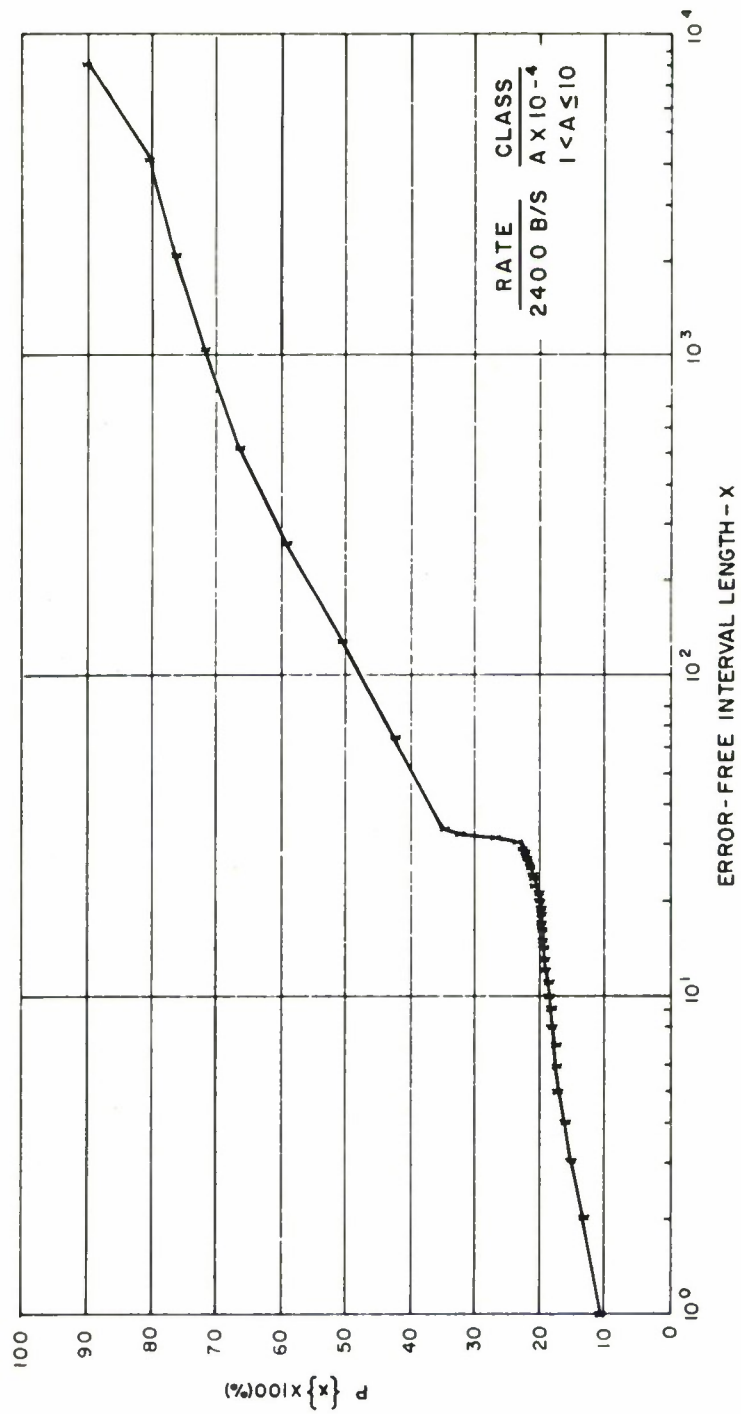


FIG.15 CUMULATIVE DISTRIBUTION FUNCTION OF ERROR FREE INTERVALS

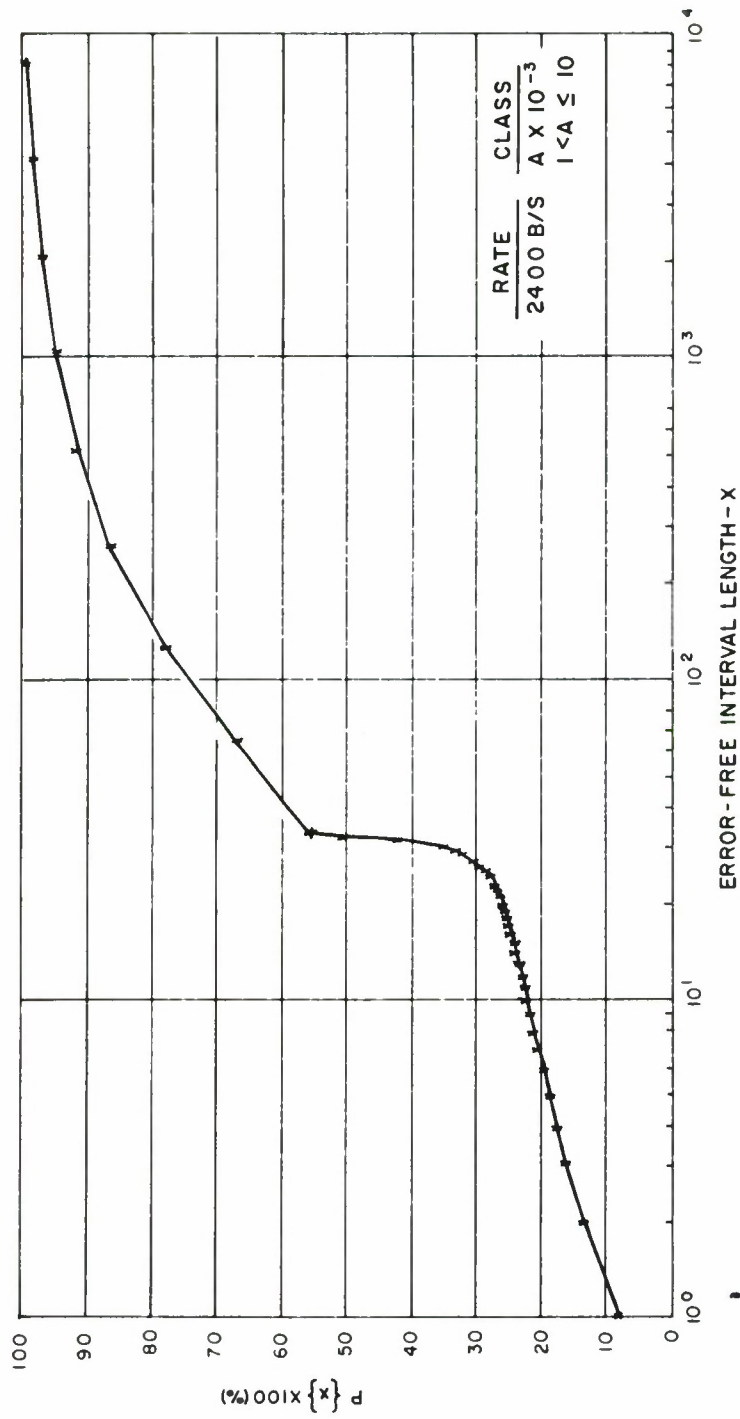


FIG.16 CUMULATIVE DISTRIBUTION FUNCTION OF ERROR FREE INTERVALS

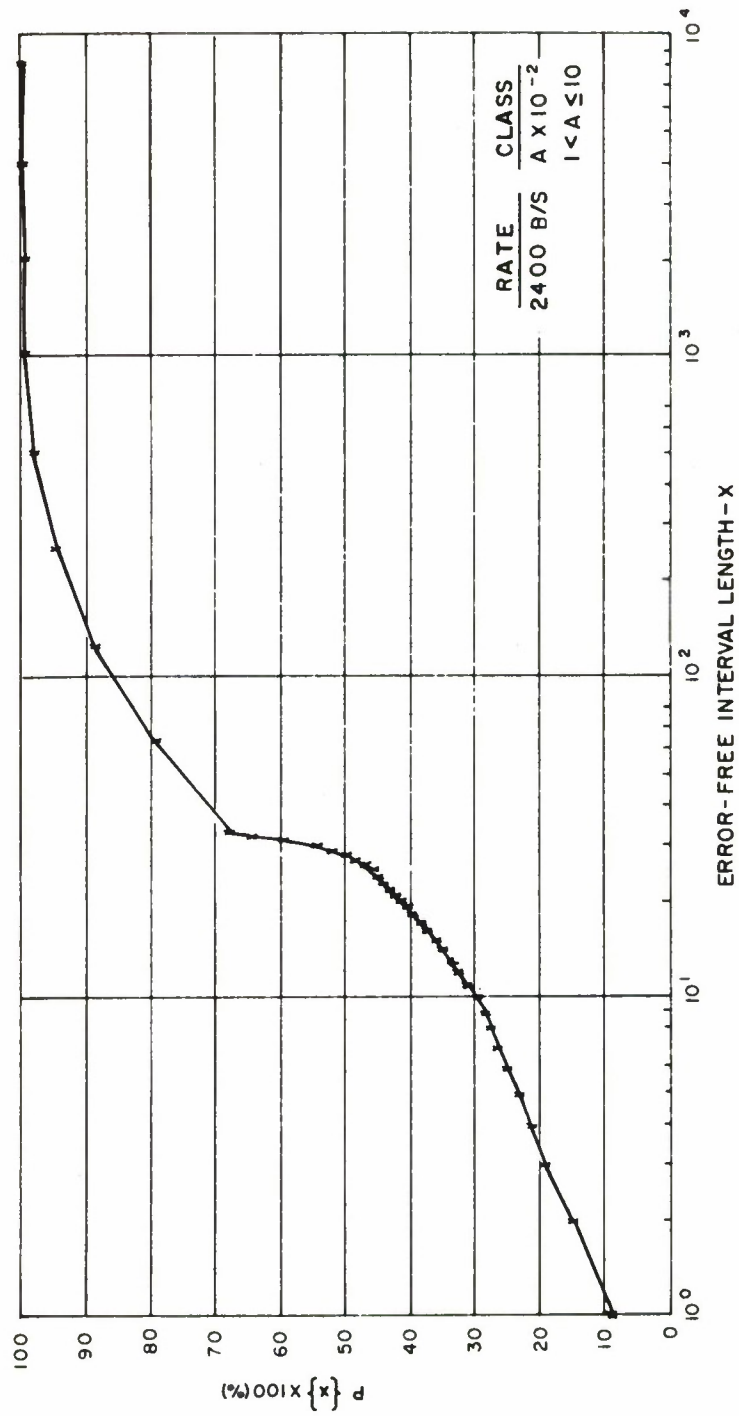


FIG.17 CUMULATIVE DISTRIBUTION FUNCTION OF ERROR FREE INTERVALS

CONSECUTIVE ERROR STATISTICS

(CLASS = 10^{-5})

(DATA RATE = 1200 bps)

SIZE	MEDIAN	STD DEV.	RANGE		MEAN
1	95.35	4.82	82.50	100.00	94.28
2	4.17	3.82	0.00	11.29	4.85
3	6.66	6.62	0.00	2.50	0.17
4	0.00	1.25	0.00	5.00	0.33
5	0.00	0.00	0.00	0.00	0.00
6	0.00	0.69	0.00	2.78	0.19

(DATA RATE = 2400 bps)

1	97.33	2.04	92.86	100.00	97.21
2	2.44	1.51	0.00	3.81	1.99
3	6.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00
5	0.00	1.68	0.00	5.36	0.59
6	0.00	0.00	0.00	0.00	0.00

TABLE I

CONSECUTIVE ERROR STATISTICS
(CLASS = 10^{-4})

(DATA RATE = 1200 bps)

SIZE	MEDIAN	STD. DEV.	RANGE		MEAN
1	95.45	2.00	88.50	98.36	94.86
2	4.30	1.45	1.39	7.24	4.45
3	0.00	2.02	0.00	11.61	0.59
4	0.00	1.08	0.00	5.94	0.30
5	0.00	0.54	0.00	2.80	0.14
6	0.00	0.33	0.00	1.92	0.06

(DATA RATE = 2400 bps)

1	97.91	5.87	78.00	99.44	94.82
2	2.09	2.60	0.56	9.37	3.46
3	0.00	0.87	0.00	3.05	0.48
4	0.00	0.77	0.00	2.65	0.43
5	0.00	0.70	0.00	2.44	0.22
6	0.00	0.48	0.00	1.63	0.19

TABLE II

CONSECUTIVE ERROR STATISTICS

(CLASS = 10^{-3})

(DATA RATE = 1200 bps)

SIZE	MEDIAN	STD. DEV.	RANGE		MEAN
1	93.31	2.62	86.30	96.69	92.52
2	6.26	2.07	3.04	12.41	6.70
3	0.37	0.57	0.00	2.12	0.62
4	0.05	0.16	0.00	0.75	0.12
5	0.00	0.03	0.00	0.09	0.01
6	0.00	0.05	0.00	0.34	0.02

(DATA RATE = 2400 bps)

1	91.23	1.72	90.67	95.15	92.51
2	7.05	1.59	4.54	8.29	6.54
3	0.60	0.29	0.23	1.18	0.67
4	0.12	0.09	0.08	0.36	0.16
5	0.03	0.06	0.00	0.17	0.04
6	0.00	0.01	0.00	0.03	0.01

TABLE III

CONSECUTIVE ERROR STATISTICS
(Class = 10^{-2})

(DATA RATE=2400 bps)					
SIZE	MEDIAN	STD. DEV.	RANGE		MEAN
1	95.73	4.04	87.66	95.73	91.70
2	9.74	2.87	4.00	9.74	6.87
3	1.91	0.83	0.24	1.91	1.07
4	0.54	0.26	0.02	0.54	0.28
5	0.10	0.05	0.00	0.10	0.05
6	0.04	0.02	0.00	0.04	0.02
(DATA RATE=1200 bps)					
1	92.34	3.57	84.72	97.95	92.13
2	7.09	2.82	1.97	12.10	6.92
3	0.49	0.66	0.07	2.34	0.75
4	0.70	0.17	0.00	0.58	0.14
5	0.00	0.05	0.00	0.17	0.04
6	0.00	0.02	0.00	0.06	0.01

TABLE IV

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10⁻⁵

DATA RATE 1200 bps

SIZE	MEDIAN	STD.DEV.	RANGL.	MEAN
1	5.12	7.63	0.0 0	27.45
2	0.00	7.30	0.00	26.42
3	0.00	3.02	0.00	8.57
4	0.00	1.05	0.00	3.92
5	0.00	3.14	0.00	12.50
6	0.00	1.32	0.00	3.92
7	0.00	0.71	0.00	2.27
8	0.00	0.86	0.00	2.72
9	0.00	0.65	0.00	1.96
10	0.00	0.70	0.00	2.17
11	0.00	0.57	0.00	2.27
12	0.00	0.00	0.00	0.00
13	0.00	0.92	0.00	2.56
14	0.00	0.00	0.00	0.00
15	0.00	1.61	0.00	4.35
16	1.96	3.78	0.00	13.64
17	1.96	2.32	0.00	6.82
18	0.00	0.69	0.00	2.78
19	0.00	0.83	0.00	2.85
20	0.00	0.67	0.00	1.96

TABLE V

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10^{-5}

DATA RATE 1200 bps

SIZE	MEDIAN	STD.DEV.	RANGE		MEAN
21	0.00	0.00	0.00	0.00	0.00
22	0.00	0.00	0.00	0.00	0.00
23	0.00	0.00	0.00	0.00	0.00
24	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00
26	0.00	0.00	0.00	0.00	0.00
27	0.00	1.38	0.00	5.55	0.37
28	0.00	0.00	0.00	0.00	0.00
29	0.00	0.00	0.00	0.00	0.00
30	0.00	0.49	0.00	1.96	0.13
31	0.00	1.53	0.00	5.88	0.62
32	0.00	1.82	0.00	5.88	1.11
33	0.00	1.32	0.00	4.35	0.72
64	1.96	2.51	0.00	8.69	2.43
128	1.88	3.48	0.00	12.82	3.14
256	1.96	2.25	0.00	7.69	2.38
512	0.00	1.76	0.00	6.52	1.29
1024	1.96	2.26	0.00	8.75	1.90
2048	3.75	2.78	0.00	7.84	3.47
4096	2.56	3.06	0.00	11.11	3.78
8192	8.69	4.42	0.00	15.22	7.91

TABLE V (Con't)

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10^{-4}

DATA RATE 1200 bps

SIZE	MEDIAN	STD.DEV.	RANGE	MEAN
1	4.98	13.05	1.60	64.61
2	0.92	4.58	0.00	25.07
3	0.94	2.36	0.00	12.40
4	0.31	1.34	0.00	6.00
5	0.18	0.77	0.00	3.49
6	0.00	0.50	0.00	1.72
7	0.00	0.58	0.00	2.41
8	0.00	0.65	0.00	3.44
9	0.00	0.53	0.00	2.58
10	0.00	0.42	0.00	1.72
11	0.00	0.30	0.00	1.07
12	0.00	0.35	0.00	1.07
13	1.16	0.47	0.00	1.72
14	0.38	0.91	0.00	3.61
15	3.06	2.07	0.00	8.23
16	5.37	3.24	0.00	12.17
17	2.58	1.48	0.00	5.82
18	0.00	0.47	0.00	1.72
19	0.00	0.51	0.00	2.04
20	0.00	0.12	0.00	0.57

TABLE VI

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10^{-4}

DATA RATE 1200 bps

SIZE	MEDIAN	STD.DEV.	MODE	MEAN
21	6.00	0.20	0.00	1.07
22	0.00	0.6	0.00	0.31
23	0.00	0.14	0.00	0.51
24	0.00	0.15	0.00	0.79
25	0.00	0.09	0.00	0.43
26	0.00	0.29	0.00	1.47
27	0.00	0.20	0.00	1.07
28	0.00	0.38	0.00	2.04
29	0.00	0.59	0.00	2.04
30	0.00	0.40	0.00	2.15
31	1.17	1.17	0.00	4.73
32	2.36	1.87	0.00	6.48
33	0.71	0.74	0.00	3.08
64	6.09	3.48	0.00	17.28
128	6.14	3.92	0.00	15.66
256	5.10	2.81	0.70	10.02
512	5.74	2.85	0.00	10.07
1024	6.04	3.74	0.00	14.29
2048	8.05	4.73	0.00	16.33
4096	9.40	5.04	0.00	16.27
8192	7.87	4.42	0.00	20.59

TABLE VI (Con't)

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10⁻³

DATA RATE 1200 bps

SIZE	MEDIAN	STD.DEV.	RANGE		MEAN
1	6.72	3.11	3.45	15.90	7.88
2	2.23	2.05	0.21	7.30	2.83
3	1.34	1.25	0.11	4.88	1.77
4	0.58	0.96	0.00	3.96	1.08
5	0.61	0.72	0.00	3.03	0.86
6	0.52	0.62	0.00	2.27	0.72
7	0.46	0.54	0.00	2.14	0.60
8	0.46	0.54	0.05	2.14	0.64
9	0.37	0.55	0.00	2.14	0.62
10	0.46	0.57	0.00	2.35	0.65
11	0.67	0.55	0.00	1.96	0.77
12	0.65	0.59	0.00	2.07	0.81
13	1.10	0.88	0.00	3.27	1.25
14	1.36	1.06	0.20	4.58	1.70
15	4.40	1.96	1.40	11.29	4.71
16	6.55	2.83	2.10	14.57	6.78
17	3.33	1.12	1.27	6.27	3.41
18	0.88	0.43	0.00	2.12	0.80
19	0.46	0.27	0.00	1.03	0.48
20	0.28	0.22	0.00	0.77	0.29

TABLE VII

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10^{-3}

DATA RATE 1200 bps

SIZE	MEDIAN	STD.DEV.	RANGE	MEAN
21	0.23	0.18	0.00	0.24
22	0.21	0.16	0.00	0.21
23	0.22	0.15	0.00	0.21
24	0.17	0.16	0.00	0.19
25	0.16	0.14	0.00	0.19
26	0.22	0.16	0.00	0.22
27	0.25	0.16	0.00	0.23
28	0.27	0.18	0.00	0.28
29	0.51	0.25	0.00	0.47
30	0.66	0.35	0.00	0.65
31	1.59	0.60	0.76	1.70
32	2.20	1.41	0.64	2.37
33	1.23	0.53	0.51	1.34
64	9.66	2.55	4.63	9.95
128	8.37	2.53	4.87	8.92
256	7.67	3.20	2.96	8.19
512	7.50	4.25	1.46	7.72
1024	5.97	4.88	0.80	7.24
2048	4.61	4.45	0.54	5.82
4096	2.31	3.03	0.00	3.38
8192	0.73	1.41	0.00	1.40

TABLE VII (Con't)

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10^{-2}

DATA RATE 1200 bps

SIZE	MEDIAN	STD.DEV.	RANGE		MEAN
1	7.82	3.88	2.10	16.41	8.21
2	3.07	2.53	1.40	10.16	4.45
3	2.43	1.92	1.04	8.31	3.11
4	1.48	1.40	0.64	5.74	2.05
5	1.17	1.11	0.52	4.70	1.73
6	1.26	0.92	0.24	3.76	1.44
7	1.08	0.75	0.29	3.13	1.34
8	1.00	0.74	0.22	2.77	1.30
9	1.10	0.67	0.33	2.74	1.29
10	1.09	0.54	0.28	2.31	1.26
11	1.23	0.58	0.42	2.37	1.36
12	1.39	0.54	0.52	2.39	1.42
13	1.87	0.58	0.80	2.85	1.86
14	2.27	0.91	0.81	4.70	2.32
15	4.27	2.75	2.95	13.35	5.55
16	9.73	3.79	3.61	16.43	8.75
17	3.60	1.71	1.84	8.33	3.90
18	1.92	0.48	0.23	2.30	1.12
19	0.84	0.30	0.21	1.35	0.83
20	0.56	0.29	0.10	1.10	0.56

TABLE VIII

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10^{-2}

DATA RATE 1200 bps

SIZE	MEDIAN	STD.DEV.	RANGE		MEAN
21	0.49	0.28	0.10	1.17	0.52
22	0.43	0.27	0.12	1.15	0.52
23	0.42	0.25	0.07	1.08	0.45
24	0.36	0.32	0.07	1.18	0.44
25	0.38	0.23	0.12	0.98	0.44
26	0.41	0.23	0.08	0.99	0.46
27	0.42	0.23	0.12	0.99	0.48
28	0.53	0.27	0.09	1.13	0.51
29	0.69	0.24	0.18	1.04	0.68
30	0.85	0.47	0.12	2.34	0.88
31	1.57	0.85	0.56	4.15	1.80
32	1.88	1.14	0.62	4.80	2.27
33	1.37	0.42	0.53	2.42	1.30
64	13.56	3.44	5.75	18.59	13.58
128	10.72	4.06	3.72	16.45	10.47
256	5.11	3.91	2.34	14.57	6.69
512	2.16	1.95	0.50	7.82	2.88
1024	1.07	0.59	0.01	1.94	1.06
2048	0.44	0.35	0.00	1.19	0.44
4096	0.43	0.22	0.00	0.71	0.18
8192	0.00	0.10	0.00	0.32	0.06

TABLE VIII (Con't)

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10^{-5}

DATA RATE 2400 bps

SIZE	MEDIAN	STD.DEV.	RANGE		MEAN
1	2.56	6.99	0.00	24.00	4.57
2	0.00	2.52	0.00	8.00	1.40
3	0.00	1.26	0.00	4.00	0.44
4	0.00	0.00	0.00	0.00	0.00
5	0.00	0.47	0.00	1.33	0.25
6	0.00	0.42	0.00	1.33	0.15
7	0.00	0.42	0.00	1.33	0.15
8	0.00	0.47	0.00	1.33	0.25
9	0.00	0.30	0.00	0.96	0.11
10	0.00	0.28	0.00	0.88	0.10
11	0.00	0.37	0.00	1.18	0.13
12	0.00	0.00	0.00	0.00	0.00
13	0.00	0.98	0.00	3.13	0.35
14	0.00	0.00	0.00	0.00	0.00
15	0.00	0.39	0.00	0.91	0.10
16	0.00	0.62	0.00	1.96	0.22
17	0.00	0.00	0.00	0.00	0.00
18	0.00	0.00	0.00	0.00	0.00
19	0.00	0.00	0.00	0.00	0.00
20	0.00	0.46	0.00	1.23	0.24

TABLE IX

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10⁻⁵

DATA RATE 2400 bps

SIZE	MEDIAN	STD. DEV.	RANGE		MEAN
21	ALL VALUES ZERO				
22	ALL VALUES ZERO				
23	0.00	0.41	0.00	1.30	0.14
24	ALL VALUES ZERO				
25	ALL VALUES ZERO				
26	ALL VALUES ZERO				
27	ALL VALUES ZERO				
28	0.00	0.28	0.00	0.88	0.10
29	0.00	0.37	0.00	1.18	0.13
30	ALL VALUES ZERO				
31	2.66	1.74	1.30	7.69	3.15
32	3.13	3.06	0.00	7.96	3.80
33	3.13	1.70	0.00	5.34	2.71
64	2.56	1.84	0.00	5.45	2.33
128	5.77	3.03	0.00	9.80	5.62
256	5.88	2.92	1.77	10.39	6.04
512	1.92	2.77	0.88	10.39	2.80
1024	2.73	2.93	0.00	9.80	3.63
2048	2.67	1.30	0.88	4.70	2.77
4096	3.64	3.55	0.00	11.50	3.85
8192	6.73	3.66	0.00	11.53	6.38

TABLE IX (Con't)

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10^{-4}

DATA RATE 2400 bps

SIZE	MEDIAN	STD.DEV.	RANGE	MEAN
1	5.63	13.70	0.55	49.80
2	2.79	2.68	0.00	9.79
3	1.67	1.93	0.00	6.73
4	0.54	0.82	0.00	2.55
5	0.82	0.65	0.00	1.84
6	0.54	0.39	0.00	1.02
7	0.35	0.35	0.00	0.71
8	0.14	0.37	0.00	1.02
9	0.00	0.42	0.00	1.27
10	0.27	0.33	0.00	1.09
11	0.00	0.39	0.00	1.36
12	0.00	0.51	0.00	1.82
13	0.23	0.26	0.00	1.27
14	0.00	0.53	0.00	1.90
15	0.10	0.34	0.00	1.18
16	0.00	0.23	0.00	0.63
17	0.00	0.39	0.00	1.36
18	0.12	0.46	0.00	1.63
19	0.10	0.21	0.00	0.73
20	0.00	0.41	0.00	1.36

TABLE X

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10^{-4}

DATA RATE 2400 bps

SIZE	MEDIAN	STD.DEV.	RANGE		MEAN
21	0.17	0.23	0.00	0.82	0.18
22	0.10	0.19	0.00	0.64	0.14
23	0.00	0.15	0.00	0.45	0.11
24	0.10	0.27	0.00	0.69	0.21
25	0.00	0.40	0.00	1.36	0.22
26	0.00	0.28	0.00	0.91	0.16
27	0.14	0.51	0.00	1.53	0.34
28	0.15	0.32	0.00	1.02	0.26
29	0.27	0.47	0.00	1.38	0.50
30	0.58	0.47	0.00	1.68	0.64
31	2.89	1.66	0.55	6.14	3.17
32	5.95	2.36	1.53	9.50	5.98
33	2.23	1.01	0.55	3.81	2.24
64	7.69	2.94	1.22	11.82	7.47
128	8.45	3.49	2.34	15.53	8.25
256	8.94	2.67	2.75	13.14	9.06
512	6.76	2.94	1.73	10.99	6.88
1024	5.03	2.67	1.63	10.50	5.47
2048	4.30	2.54	1.36	8.43	4.11
4096	3.23	2.89	1.22	9.80	4.40
8192	4.86	3.53	1.18	12.71	5.34

TABLE X. (Con't)

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10^{-3}

DATA RATE 2400 bps

SIZE	MEDIAN	STD.DEV.	RANGE		MEAN
1	9.32	1.99	4.98	11.35	8.73
2	3.47	2.61	2.81	10.56	4.66
3	1.98	1.57	1.28	5.97	2.73
4	1.31	0.68	0.38	2.14	1.27
5	1.24	0.66	0.33	2.50	1.35
6	1.17	0.65	0.29	2.46	1.19
7	0.85	0.43	0.27	1.70	0.94
8	0.48	0.29	0.19	1.21	0.58
9	0.50	0.32	0.16	1.18	0.55
10	0.32	0.29	0.08	1.02	0.40
11	0.35	0.46	0.03	1.49	0.43
12	0.51	0.45	0.03	1.43	0.48
13	0.37	0.45	0.00	1.40	0.44
14	0.51	0.43	0.00	1.24	0.45
15	0.32	0.40	0.00	1.03	0.40
16	0.27	0.50	0.00	1.43	0.46
17	0.37	0.33	0.00	0.95	0.35
18	0.41	0.36	0.00	1.11	0.40
19	0.32	0.43	0.00	1.36	0.39
20	0.30	0.37	0.00	1.23	0.40

TABLE XI

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10^{-3}

DATA RATE 2400 bps

SIZE	MEDIAN	STD.DEV.	RANGE		MEAN
21	0.19	0.39	0.77	1.03	0.32
22	0.27	0.17	0.12	0.58	0.35
23	0.48	0.21	0.14	0.74	0.50
24	0.54	0.40	0.19	1.24	0.69
25	0.59	0.45	0.16	1.63	0.72
26	1.08	0.43	0.29	1.45	0.94
27	0.74	0.59	0.16	2.07	0.85
28	0.68	0.67	0.38	2.45	0.90
29	1.36	0.92	0.88	3.51	1.66
30	2.06	1.04	1.39	4.69	2.34
31	4.84	3.29	1.70	11.76	6.58
32	7.59	3.56	2.79	15.19	8.40
33	3.50	1.85	2.24	6.97	4.51
64	11.52	2.35	8.23	15.06	11.88
128	11.48	2.34	8.13	15.09	11.33
256	7.92	2.38	4.52	10.98	8.00
512	4.75	2.89	2.59	10.88	5.51
1024	1.97	2.05	1.16	7.24	2.88
2048	1.29	2.12	0.54	6.87	2.17
4096	1.23	1.23	0.40	4.15	1.47
8192	0.74	0.50	0.27	1.73	0.91

TABLE XI (Con't)

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10⁻²

DATA RATE 2400 bps

SIZE	MEDIAN	STD. DEV.	RANGE		MEAN
1	13.65	4.65	4.36	13.65	9.00
2	7.13	1.11	4.90	7.13	6.01
3	4.07	0.09	3.89	4.07	3.98
4	2.82	0.13	2.56	2.82	2.69
5	2.08	0.17	1.74	2.08	1.91
6	1.73	0.30	1.13	1.73	1.43
7	1.58	0.27	1.03	1.58	1.31
8	1.50	0.32	0.86	1.50	1..8
9	1.70	0.43	0.83	1.70	1.27
10	1.63	0.41	0.81	1.63	1.22
11	1.78	0.57	0.63	1.78	1.20
12	1.84	0.66	0.51	1.84	1.18
13	1.75	0.55	0.69	1.79	1.24
14	1.79	0.54	0.72	1.79	1.25
15	1.76	0.56	0.64	1.76	1.20
16	2.00	0.72	0.56	2.00	1.28
17	1.60	0.47	0.66	1.60	1.13
18	1.55	0.45	0.66	1.55	1.10
19	1.50	0.48	0.54	1.50	1.50
20	1.40	0.45	0.49	1.40	0.95

TABLE XII

ERROR FREE INTERVAL STATISTICS

ERROR CLASS 10⁻²

DATA RATE 2400 bps

SIZE	MEDIAN	STD.DEV.	RANGE		MEAN
21	1.17	0.26	0.64	1.17	0.91
22	0.92	0.11	0.70	0.92	0.81
23	0.87	0.02	0.82	0.87	0.85
24	0.78	0.00	0.77	0.78	0.77
25	1.00	0.09	0.82	1.01	0.91
26	1.19	0.18	0.83	1.19	1.01
27	2.04	0.55	0.94	2.04	1.49
28	2.19	0.51	1.16	2.19	1.68
29	2.62	0.50	1.62	2.62	2.12
30	2.71	0.46	1.79	2.71	2.25
31	5.12	0.28	4.55	5.12	4.83
32	6.95	1.53	3.89	6.95	5.42
33	3.59	0.22	3.16	3.59	3.38
64	12.88	1.25	10.39	12.88	11.64
128	10.19	0.85	8.48	10.19	9.34
256	6.28	0.30	5.68	6.28	5.98
512	3.17	0.12	2.93	3.17	3.05
1024	1.35	0.08	1.19	1.35	1.27
2048	0.65	0.14	0.36	0.65	0.51
4096	0.27	0.12	0.04	0.27	0.16
8192	0.10	0.05	0.00	0.10	0.05

TABLE XII (Con't)

Unclassified

Security Classification

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1. ORIGINATING ACTIVITY (Corporate author) The MITRE Corporation Bedford, Massachusetts		2a. REPORT SECURITY CLASSIFICATION Unclassified	
		2b. GROUP	
3. REPORT TITLE HF Channel Data Error Statistics Description (I)			
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10. AVAILABILITY/LIMITATION NOTICES Distribution of this document is unlimited.			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY Directorate of Aerospace Instrumentation, Electronic Systems Division, L. G. Hanscom Field, Bedford, Massachusetts.	
13. ABSTRACT A statistical description is presented of errors observed on a typical HF data transmission channel. The description includes the probability distribution functions for consecutive errors and error-free gaps, and their associated cross-density statistics.			

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
SYSTEMS AND MECHANISMS Data Transmission Systems Multichannel Radio Systems Voice Communication (HF) Systems INFORMATION THEORY Coding MATHEMATICS Statistical Analysis, HF Error Locations Statistical Distributions, HF Error Locations Statistical Data, HF Error Locations						

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